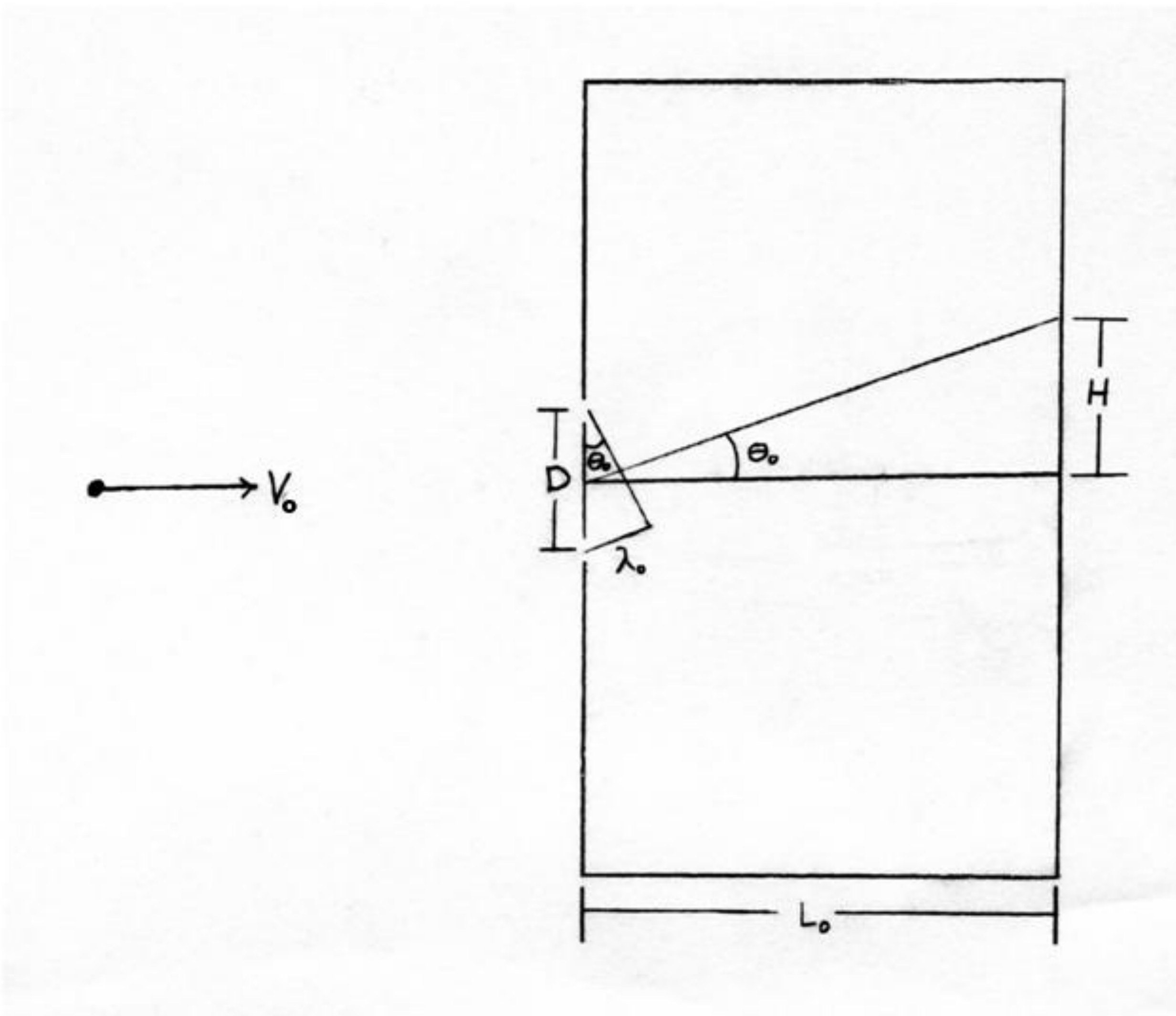
Double Slit Thought Experiment

Today we are going to take a walk in the lower hills of the math world on a side path where we have to hack through brush to see the sights. A peek will be taken of the double slit experiment. This is something well known and not too difficult to understand. The experiment is usually done with light. Here it will be done with electrons.

The double slit thought experiment will have electrons aimed at two slits in a box. The back of the box will be covered with a material that responds to being struck by electrons, thus recording the electron impact. This is shown in Figure 1.

Figure 1



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The electron rest mass is $m_0 = 9.10939 \times 10^{-28}\ gm.$

Planck's constant is $h = 6.62607 \times 10^{-27} erg\ sec.$

The speed of light in a vacuum is $c = 2.99792 \times 10^{10}\ cm/sec.$

The wavelength of the electron is given by

$$\lambda_0 = \frac{h}{m_0 v_0} \times \sqrt{1 - \frac{v_0^2}{c^2}}$$

Take the speed of the electron to be $v_0 = 0.4c.$

The wavelength of electrons traveling at $0.4c$ is then

$$\boxed{\lambda_0 = 5.56867 \times 10^{-10}\ cm}$$

Set $D = 3 \times 10^{-9} cm$ and $L_0 = 10\ cm$.

Then

$$\boxed{\theta_0 = \sin^{-1}\left(\frac{\lambda_0}{D}\right) = 10.6974\ degrees}$$

H is the distance of the first maximum fringe above the horizontal and is determined by what is known, i.e. determined by $L_0, D, \lambda_0\ and\ v_{e_0}$. Using simple trigonometry

$$\sin(\theta_0) = \frac{\lambda_0}{D} = \frac{H}{\sqrt{H^2 + L_o^2}}$$

$$\Longrightarrow H = \frac{L_o}{\sqrt{\left(\frac{D}{\lambda_0}\right)^2 - 1}}$$

$$\boxed{\Longrightarrow H = 1.88905\ cm}$$

The first maximum above the horizontal at the back of the box is at $1.88905\ cm$.

On a planet far off in the galaxy, small traces of oxygen have been found in the atmosphere pointing to at least a green slim on the surface. The British are sending an expedition to the planet knowing that through Darwinian evolution by the time they get there that green slim will be a population of intelligent beings whose resources can be exploited and the populace can be brought into the British Empire. The main problem with the expedition is boredom. Being from England, none of them speak English properly to where they can be understood. When they are understood the topic is the weather, which changes little in their spacecraft, or whether their spacecraft is flat or round or about the size of the Empire and how it should encompass the whole universe. To relieve the boredom and have a little fun they will do the electron double slit experiment. The only problem is that the equipment to do the experiment was left on earth and they are moving at two-tenths the speed of light relative to earth. At least they are traveling in the same direction that the electrons are moving toward the double slits. So they will with their measuring system view the experiment and of course come up with the same result on the screen at the back of the box.

Their speed will be $V = 0.2c$. They will be interested in determining the angle, θ, of the electron after it goes through the double slits. There are several methods that can be use to get the angle and of course, they will all agree.

The first method is to determine the wavelength of the electron in the frame where the experiment is moving and use the equations above to give the angle being looked for.

Method 1

As measured by those on their way to visit the green slim, the velocity of the electron is given by

$$v = \frac{v_0 - V}{1 - \frac{v_0 V}{c^2}}$$

$$\boxed{v = 0.217391\ c}$$

The momentum is given by

$$\begin{aligned} p &= mv \\ &= m_0 \gamma v \\ &= \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} v \end{aligned}$$

$$\boxed{p = 6.07210 \times 10^{-18} \frac{gm - cm}{sec}}$$

The wavelength is given by

$$\boxed{\lambda = \frac{h}{p} = 1.09123 \times 10^{-9} cm}$$

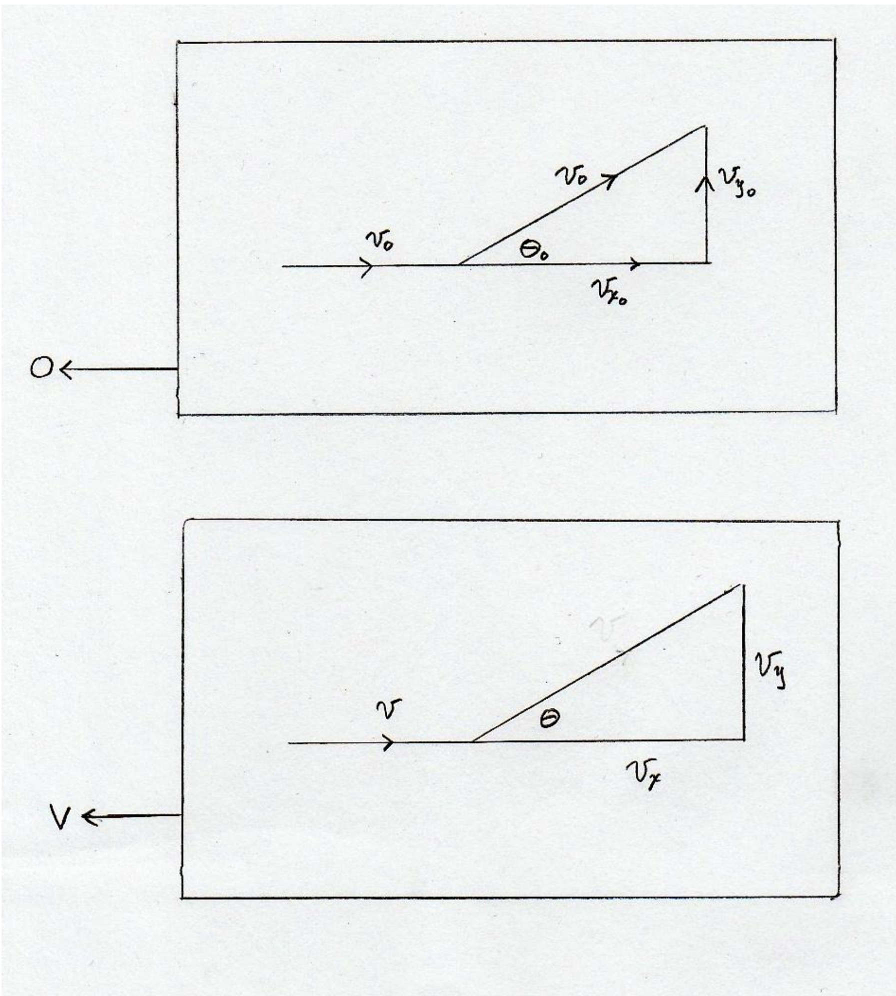
The angle is given by

$$\boxed{\theta = \sin^{-1}\left(\frac{\lambda}{D}\right) = 21.3303\ degrees}$$

There is an assumption that the speed of the electron is the same after going through the slits as it was before going through. Does the electron just go through the slits or is there an encounter between the electron and the slits. A little fun was had with this in an earlier hike.

Method 2

The angle can be found by another method. The figure below shows the path of the electron in the two frames.



$$v_0 = 0.400000\, c$$

$$\theta_0 = \sin^{-1}\left(\frac{\lambda_0}{D}\right) = 10.6974\, degrees$$

$$v_{x_0} = v_0 \cos(\theta_0) = (0.4c)\cos(10.7^o) = \ 0.393048 \text{ c}$$

$$v_{y_0} = v_0 \sin(\theta_0) = (0.4c)\sin(10.7^o) = 0.0742489 \text{ c}$$

Equations for v_x and v_y are.

$$v_x = \frac{v_{x_0} - V}{1 - \frac{v_{x_0} V}{c^2}} = 0.209519\ c$$

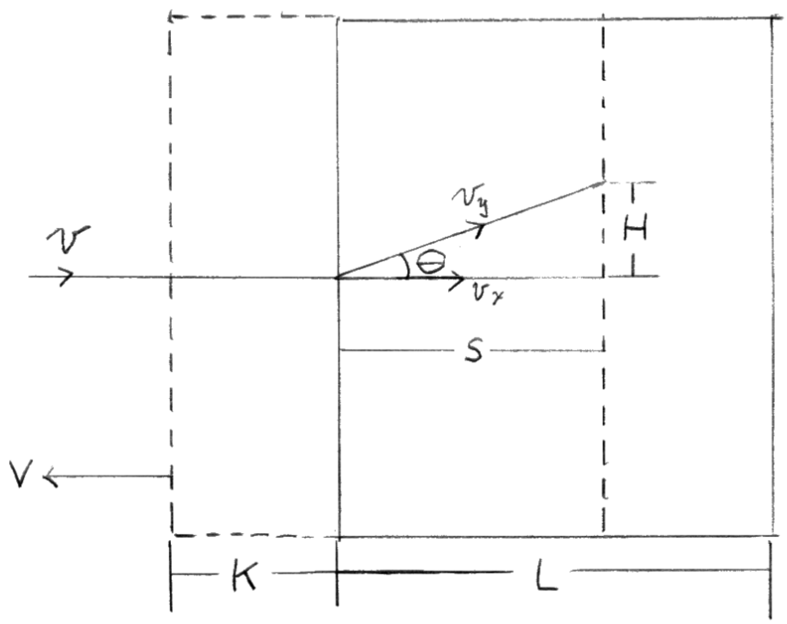
$$v_y = \frac{\sqrt{1 - \frac{V^2}{c^2}} \times v_{y_0}}{1 - \frac{v_{x_0} V}{c^2}} = 0.0789554\ c$$

$$\theta = tan^{-1}\left(\frac{v_y}{v_x}\right) = \ 20.6485\ degrees$$

Method 2 is in disagreement with Method 1 that was based on wavelength. Luckily there is a third method that has to be correct and should verify method 1.

The third method should be foolproof. No matter what inertial frame one is in, there is one thing everyone can agree on. When everyone stops moving around and gets together in the frame in which the box with a screen is at rest, they will all agree as to where on the screen in the back of the box the electron hit. Outside of the Twilight Zone those standing together cannot be looking at the screen and seeing the mark the electron made in different places. They will all agree on the value of H.

Method 3.



The angle θ can be determined from $H\ and\ S$. H is known. S can be found with a little math that a mathdrooler can understand.

$$S = v_x \times t$$

$$\Rightarrow t = \frac{s}{v_x}$$

$$K = V \times t = V \times \frac{s}{v_x}$$

$$S = L - K \ = L_0\sqrt{1 - \frac{V^2}{c^2}} - \frac{V}{v_x} \times S$$

$$\Rightarrow S\left(1 + \frac{V}{v_x}\right) = L_0\sqrt{1 - \frac{V^2}{c^2}}$$

$$\Longrightarrow S = \frac{L_0\sqrt{1-\frac{V^2}{c^2}}}{\left(1+\frac{V}{v_x}\right)}$$

$$S = \frac{10\sqrt{1-.2^2}}{1+\frac{0.2}{0.21}}\, cm \ = \ 5.01284\ cm$$

$$\boxed{\theta = \tan^{-1}\left(\frac{H}{S}\right) = \ 20.6485\ degree}$$

Methods 2 and 3 give the same result for the angle to six places. Method 1 gives a slightly higher value for the angle. Method 1 relied on the relativistic wavelength of the electron; Method 2 relied on the relativistic transformation of the electrons velocity while Method 3 relied on the motion of the box.

The wavelength that would give each of the results can be found from

$$\lambda = D \times \sin\,(\theta)$$

This gives

$$\lambda_1 = 3\times 10^{-9}\ cm\ \times \sin(21.3303\ degrees)$$
$$= \ 1.09123 \times 10^{-9}\ cm$$

$$\lambda_2 = 3\times 10^{-9}\ cm\ \times \sin(20.6485\ degrees)$$
$$= \ 1.05790 \times 10^{-9}\ cm$$

$$\lambda_3 = 3\times 10^{-9}\ cm\ \times \sin(20.6485\ \ degrees)$$
$$= \ 1.05790 \times 10^{-9}\ cm$$

λ_1 is the relativistic wavelength of the incoming electron. How is $\lambda_2\ and\ \lambda_3$ interpreted?

A little more investigation may give a better understanding of what is real, at least in our walk in the lower hills of the math world.

The Englishmen were moving toward the planet with the slim that by Darwinian evolution will be new peoples to exploit by the time they get there at $V = .4c$. Consider their journey at other speeds. Below is the wavelength of the electron after encountering the double slits measured by the Englishmen traveling at speeds of $V = 0\ to\ .4c$.

Figure 4

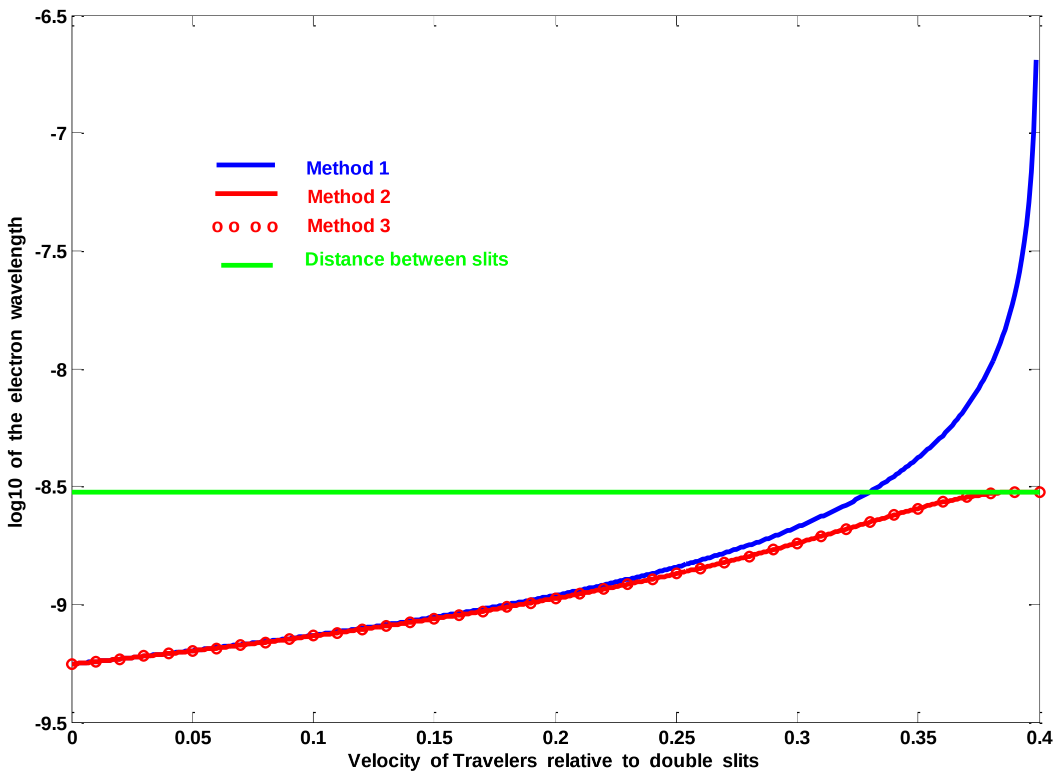


Figure 4 shows the velocity of the travelers, V, plotted against $\log_{10}$ of the wavelength of the electron as measured by the travelers. The log scale was use for a better visual display. For example, -9 on the vertical axis is $10^{-9}\ cm$.

The green horizontal line is the distance between the double slits. Where the blue curve crosses the green line the wavelength of the electron becomes longer than the distance between the slits. It would be unlikely that the electron would be seen on the screen. But all agree that the electron did contact the screen at the first maximum above the horizontal. Method 1 where the wavelength of the electron is the same on the front side of the slits as it is on the back side of the slits is what is used in all lower lever physics for the double slip experiment. It cannot be used by the travelers here.

Method 3 is based on physically where the electron contacted the screen, which all agree to. Method 2 is based on the relativistic transformation of the electrons motion in the rest frame. It is a curiosity that Methods 2 and 3 are in total agreement.

Method 1 in Figure 4, the blue line, is above Methods 2 and 3, the red line and red dots. If methods 2 and 3 are correct, then the electron wavelength is less on the backside of the slits than it is on the front side. The electron gained energy after encountering the slits as measured by the travelers. The box the slits are in then must have lost energy. So there was an interaction between the electron and the box. Of course, the energy and the momentum of the electron before encountering the slits will be given to the box when the electron strikes the screen. If the back side with the screen were removed, the travelers would see the electron speed up and the box slow down just from the electron passing the slits. In the rest frame one does not see the electron change wavelength and there is no transfer of energy from the electron to the box. So Methods 2 and 3 cannot be valid at rest and Method 1 cannot be valid in motion.

All this confusion is only for us mathdroolers who hike the lower hills of the math world. To those who hike the high mountains and peaks this is all child's play and easy to understand and explain. Having less or no understanding, one can fantasize other realities.

Let's say that the electron or any particle has no wavelength. Let's say that the wavelength is a property of or describes the encounter of two entities. It can be called an encounter wave. Consider Method 2 to find the wavelength as a function of the known quantities.

λ is to be found as a function of $V \text{ and } v_0$.

$$\lambda = \lambda(V, v_0)$$

To start, recall

$$v_{x_0} = v_0 \frac{\sqrt{D^2 - \lambda_0^2}}{D} \quad and \quad v_{y_0} = v_0 \frac{\lambda_0}{D}$$

$$v_x = \frac{v_{x_0} - V}{1 - \frac{v_{x_0} V}{c^2}} \quad and \quad v_y = \frac{\sqrt{1 - \frac{V^2}{c^2}} \times v_{y_0}}{1 - \frac{v_{x_0} V}{c^2}}$$

Now for the fun part

$$\lambda_0(v_0, m_0) = \frac{h}{m_0 v_0 \gamma} = \frac{h}{m_0 v_0}\sqrt{1 - \frac{v_0^2}{c^2}}$$

$$\lambda = Dsin(\theta) = \frac{v_y}{\sqrt{v_x^2 + v_y^2}}$$

$$= D\frac{\sqrt{1 - \frac{V^2}{c^2}} \times v_{y_0}}{\sqrt{\left(v_{x_0} - V\right)^2 + \left(1 - \frac{V^2}{c^2}\right) v_{y_0}^2}}$$

$$= D\frac{\sqrt{1 - \frac{V^2}{c^2}} \times \frac{\lambda_0}{D} v_0}{\sqrt{\left(v_0 \times \sqrt{\frac{\sqrt{D^2 - \lambda_0^2}}{D}} - V\right)^2 + \left(1 - \frac{V^2}{c^2}\right)\left(\frac{\lambda_0}{D} v_0\right)^2}}$$

$$= D\frac{\sqrt{1 - \frac{V^2}{c^2}} \times \frac{\lambda_0}{D}}{\sqrt{\left(\sqrt{1 - \left(\frac{\lambda_0}{D}\right)^2} - \frac{V}{v_0}\right)^2 + \left(1 - \frac{V^2}{c^2}\right)\left(\frac{\lambda_0}{D}\right)^2}}$$

$$\boxed{\lambda = \lambda(V, v_0) = \frac{D}{\sqrt{\frac{\left(\sqrt{1 - \left(\frac{\lambda_0}{D}\right)^2} - \frac{V}{v_0}\right)^2}{\left(1 - \frac{V^2}{c^2}\right)\left(\frac{\lambda_0}{D}\right)^2} + 1}} \qquad where \quad \lambda_0(v_0) = \frac{h}{m_0 v_0}\sqrt{1 - \frac{v_0^2}{c^2}}}$$

The same can be done from Method 3. Not going through the details and just giving the result one has:

$$\lambda = \lambda(V, v_0) = D\frac{H(v_0)}{\sqrt{S^2(V, v_0) + H^2(v_0)}}$$

$$where \quad H(v_0) = L_0\frac{\lambda_0(v_0)}{\sqrt{D^2 - \lambda_0^2(v_0)}}$$

$$and \quad S(V, v_0) = L_0\frac{\sqrt{1 - \left(\frac{V}{c}\right)^2}}{1 + \frac{V}{v_0\sqrt{1 + \left(\frac{\lambda_0}{D}\right)^2}}}$$

$$with \quad \lambda_0(v_0) = \frac{h}{m_0 v_0}\sqrt{1 - \frac{v_0^2}{c^2}}$$

S in a more pleasing form can written

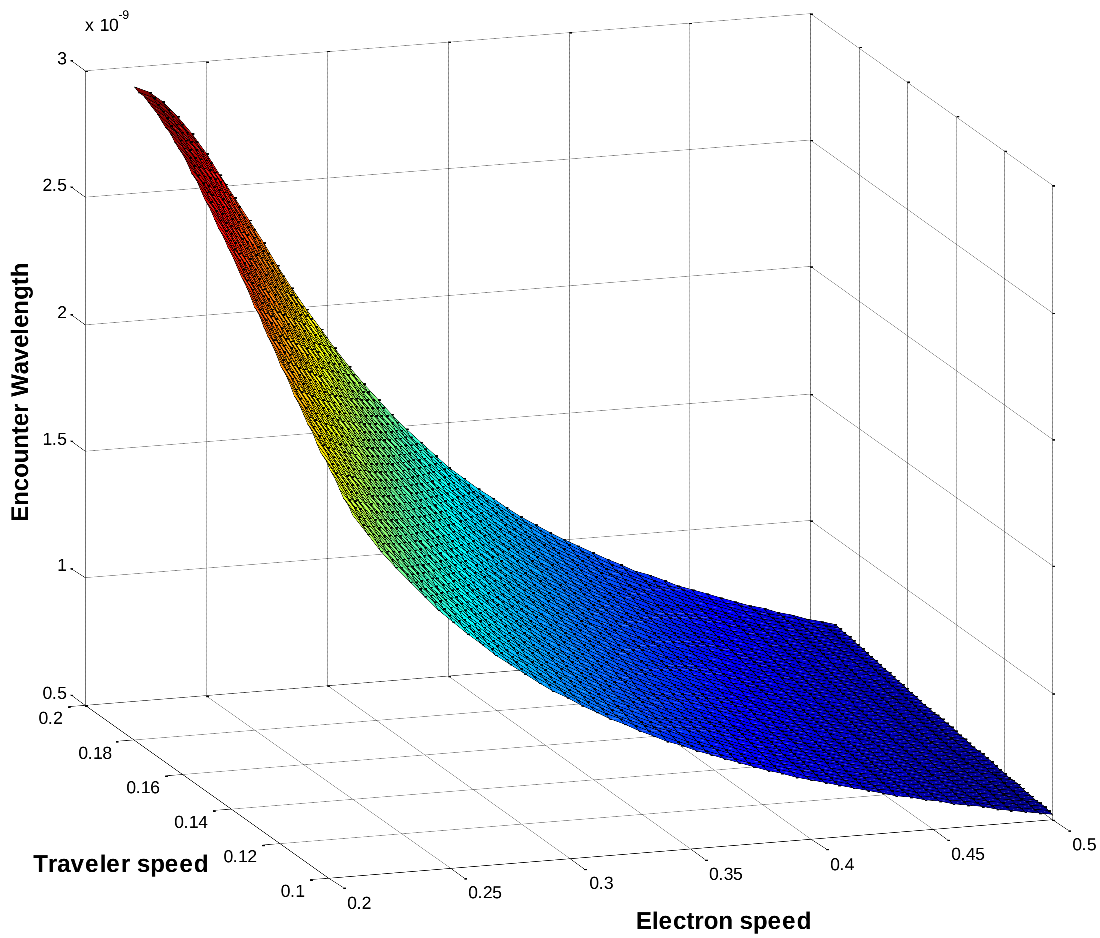
$$S = S(V, v_0) = L_0\alpha\left(1 - \beta\frac{V}{v_0}\right) \quad where$$

$$\alpha(V) = \frac{1}{\sqrt{1 - \left(\frac{V}{c}\right)^2}}$$

$$\beta\big(\lambda_0(v_0)\big) = \frac{1}{\sqrt{1 - \left(\frac{\lambda_0}{D}\right)^2}}$$

$$\lambda_0(v_0) = \frac{h}{m_0 v_0}\sqrt{1 - \frac{v_0^2}{c^2}}$$

Before leaving the lower hills of the math world a surface oplot of the encounter wavelength verses the speed of the electron and that of the travelers would be entertaining. The range of speeds will be such that the speed of the travelers is no greater than that of the electron. Otherwise, they would see (by see is meant measure) the electron moving away from them and the box with the slits moving also away but at a greater speed. This could be looked at but it is getting late. The speeds are given in units of the speed of light.



The wavelength gets shorter as the speed of the electron increases as expected. The wavelength gets longer as the speed of the travelers increases as expected.

This ends the hike today in the lower hills of the math world. It was somewhat long and a little strenuous. The sun is setting and one does not want to be caught out in the dart in the math world.

Mathdrooler 15 May 2019