Feedback Fun

Today is a good day for a trek in the lower hills of the math world. Even easy level hikes can have interested views. Consider a number n composed of two prime factors a and b. One trail to take with the relation $n = ab$ is

$$n = ab$$
$$n = ab - a^2 + a^2$$
$$n = a(b - a) + a^2$$

$$(1) \qquad \Rightarrow a = \sqrt{n - a(b - a)}$$

Another avenue is

$$n = ab - b^2 + b^2$$

$$(2) \qquad \Rightarrow b = \sqrt{n - b(a - b)}$$

This does not appear to be too exciting and does not solve for either a or b, both of which appear on both sides of the equal sign. A term being on both sides of the equal sign leads one to that which moves all nature and is of great beauty in the lower hills of the math world, iterative feedback. Start with some a and b, slove for a in (1), put that a and b into (2), solve for b and repeat the process.

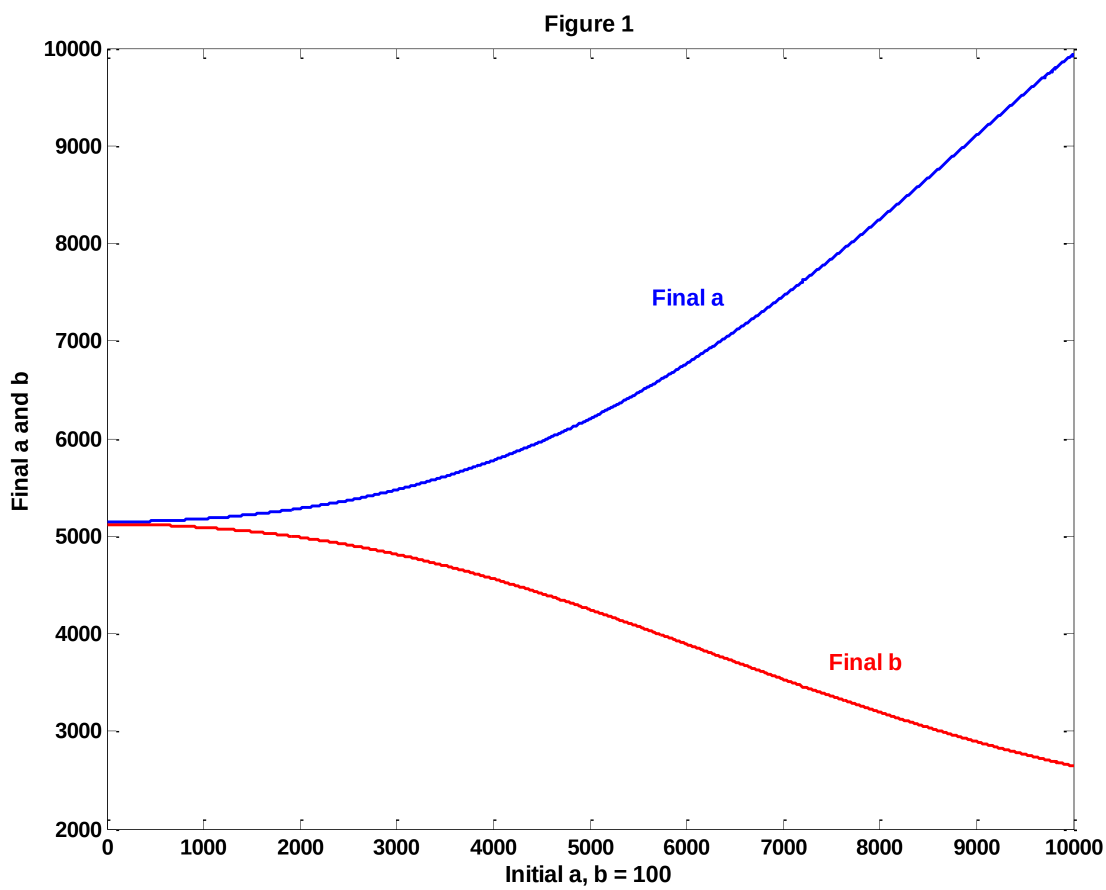
$$\rightarrow\rightarrow a = \sqrt{n - a(b - a)}$$
$$\uparrow \qquad \downarrow$$
$$\leftarrow\leftarrow b = \sqrt{n - b(a - b)}$$

Feedback is of interest when done with real numbers. Strange landscapes like chaos and fractals can be seen in it. When done with integers, feedback is boring. Once an integer value repeats itself the sequence will repeat. However, integers are what is of interest on this hike. So the values on the right side of the equal signs will be rounded to integers. To make things a little more interesting the a value will be rounded down and the b value will be rounded up. The process becomes

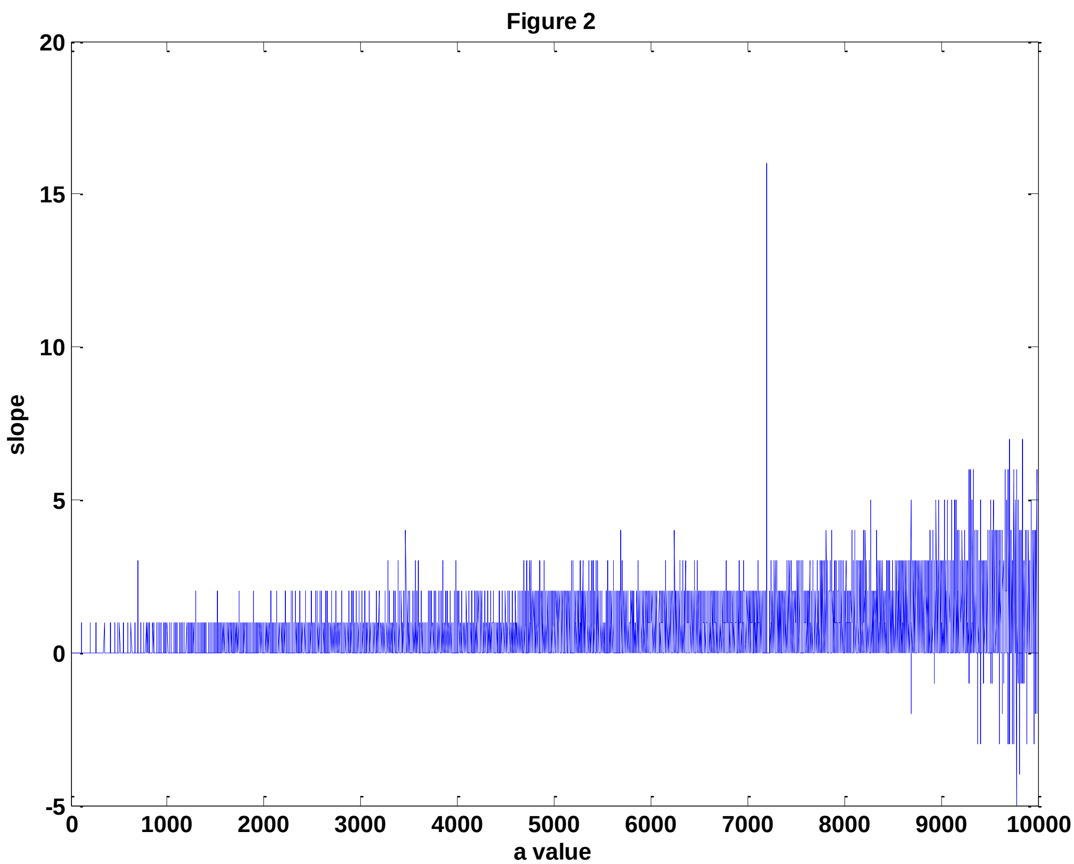
$$FB1(feedback\ 1)$$

$$\begin{array}{l} \rightarrow\rightarrow a = floor\left(\sqrt{n - a(b-a)}\right) \\ \uparrow \qquad \downarrow \\ \leftarrow\leftarrow b = ceil\left(\sqrt{n - b(a-b)}\right) \end{array}$$

There are three decisions to make: a and b and the number of iterations. Take n to be some large number, say $n = 26345797$. To start with b will be set to 100, a will very from $1\ to\ 10{,}000$ and 20 iterations will be made. The plot below shows the final $a\ and\ b$ after 20 interations plotted against the initial a.

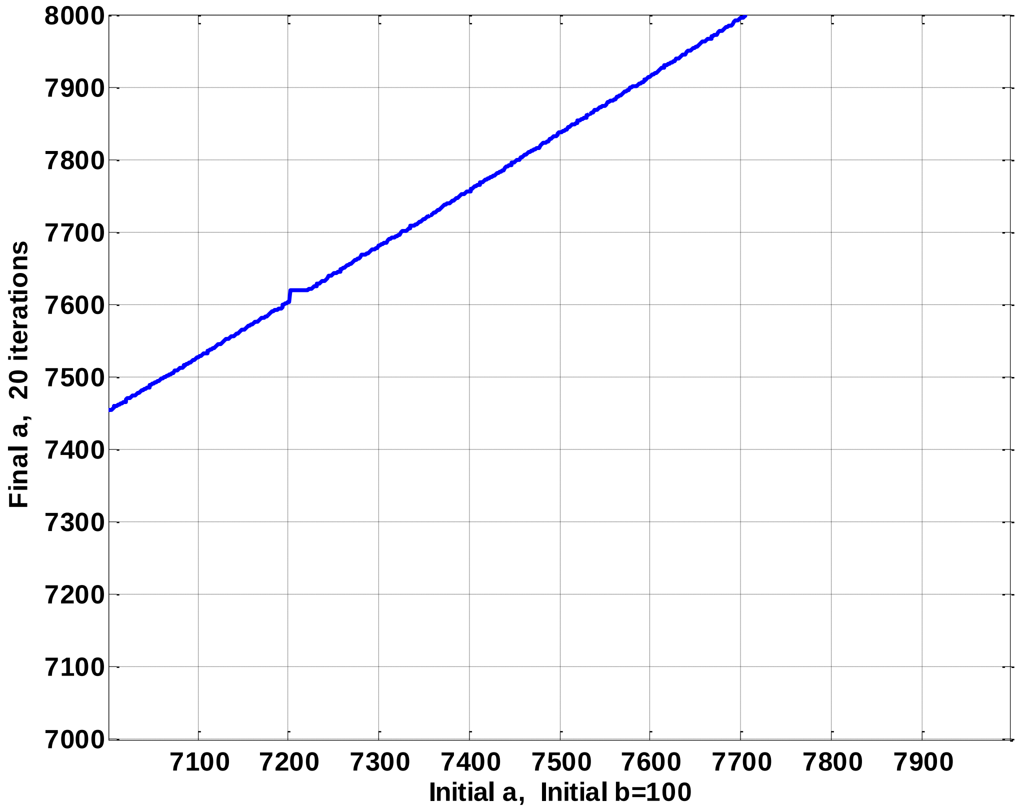


The curves for a and b are shapely but not of much interest. They look a little jagged, which would be expected from the rounding process. Plotting the slope of the curve for a, Figure 2, should show the jaggedness.

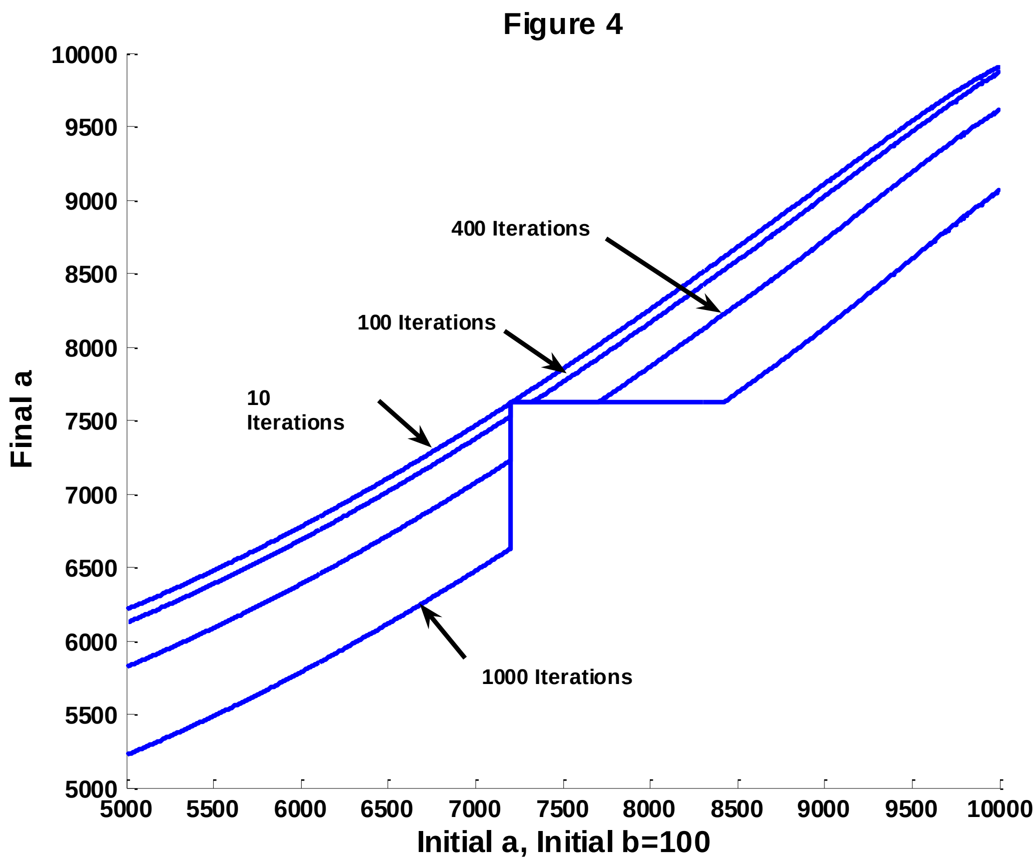


The jaggedness is there as expected, but there seems to also be an anomaly present. There is a spike between $intial\ a$ from 7000 to 8000. $Final\ a$ plotted against $Initial\ a$ in this interval is shown in Figure 3.

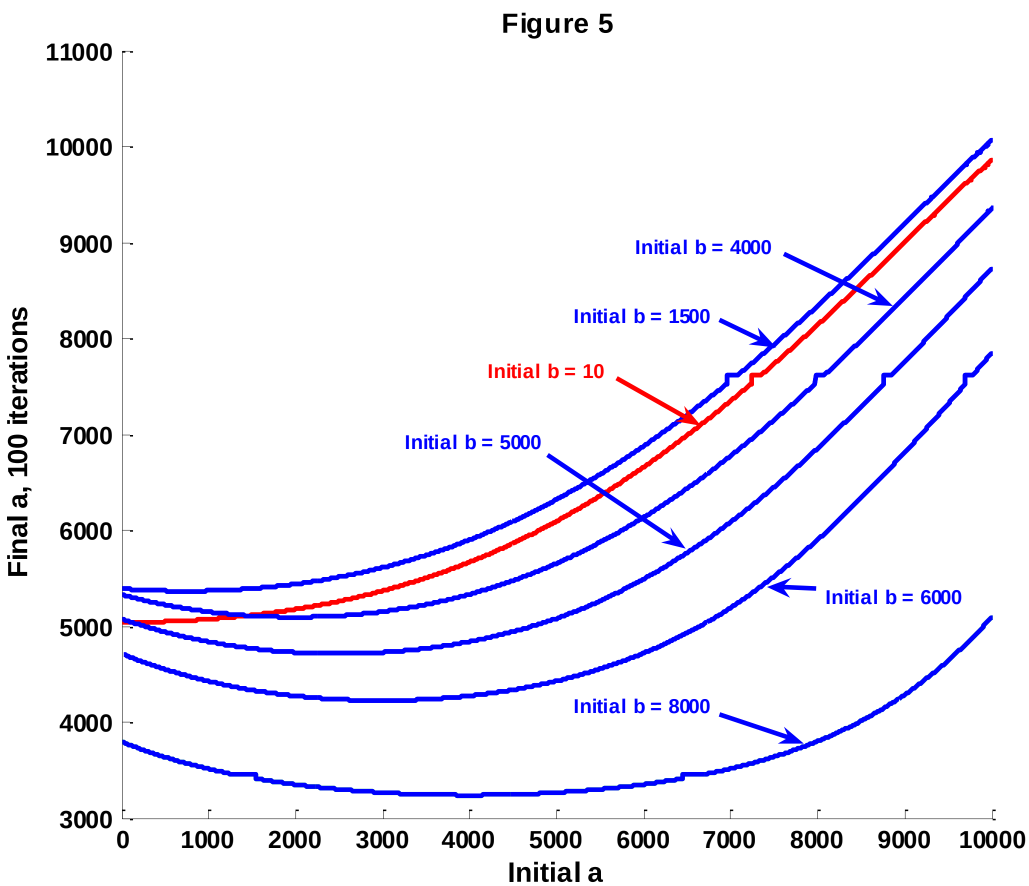
Figure 3



A step is seen from around 7200 to 7220. This may be an artifact of the low number of iterations taken. Taking 10, 100, 400 and 1000 iterations, an expanded area is shown in Figure 4.

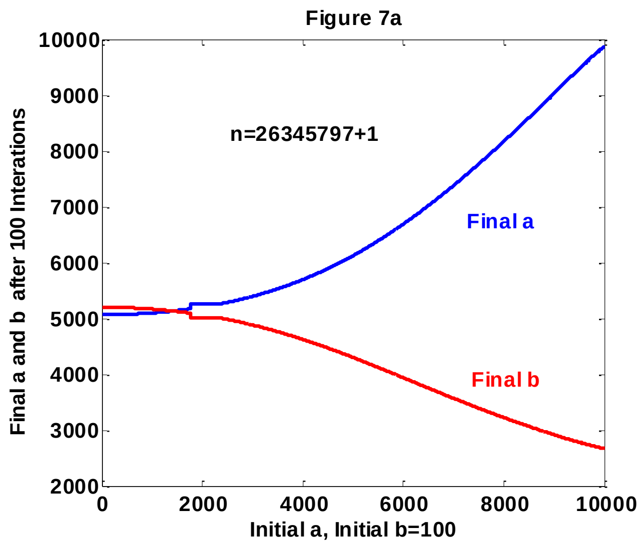


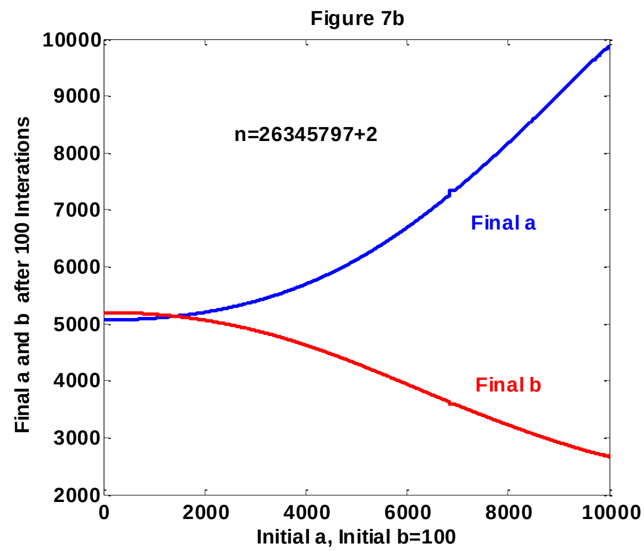
The step is more pronounced as the iterations increases. It is as if a signal is coming out of noise. The more samples taken, in this case iterations made, the less the "noise" and the more pronounced the signal. The value of the initial b is 100. What if the initial b was taken at other values? Figure 5 shows the final a for several initial values of b.

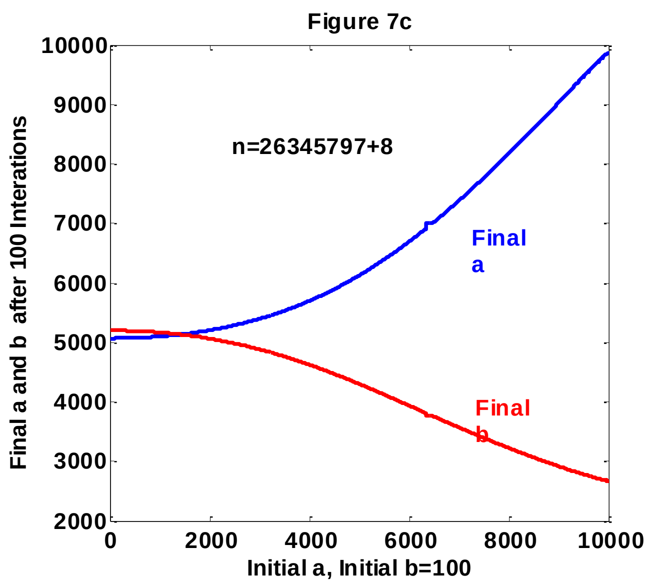


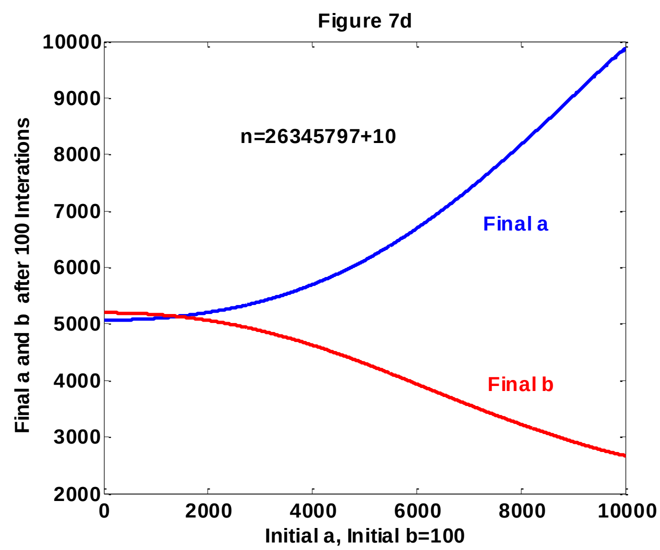
From the starting place of initial $b = 10$, the step moves to the left until about $b = 1500.$ It then moves to the right at least up to initial $b = 6000$ an likely beyond. At initial $b = 8000$ something strange happens. There are two steps, one at about initial $a = 1500$ and one at about initial $a = 6500$. The steps are facing each other. The height of the steps appears to be the same as does the height of the steps for initial $b < 6000$. Although this trail may lead further with interesting sights, with the time remaining other trails need to be transversed.

The number being investigated is $n = 26345797$. What happens when this number is changed by adding 1 to it? The figures below are for various slight changes to n taking the initial $b = 100$ and 100 iterations.



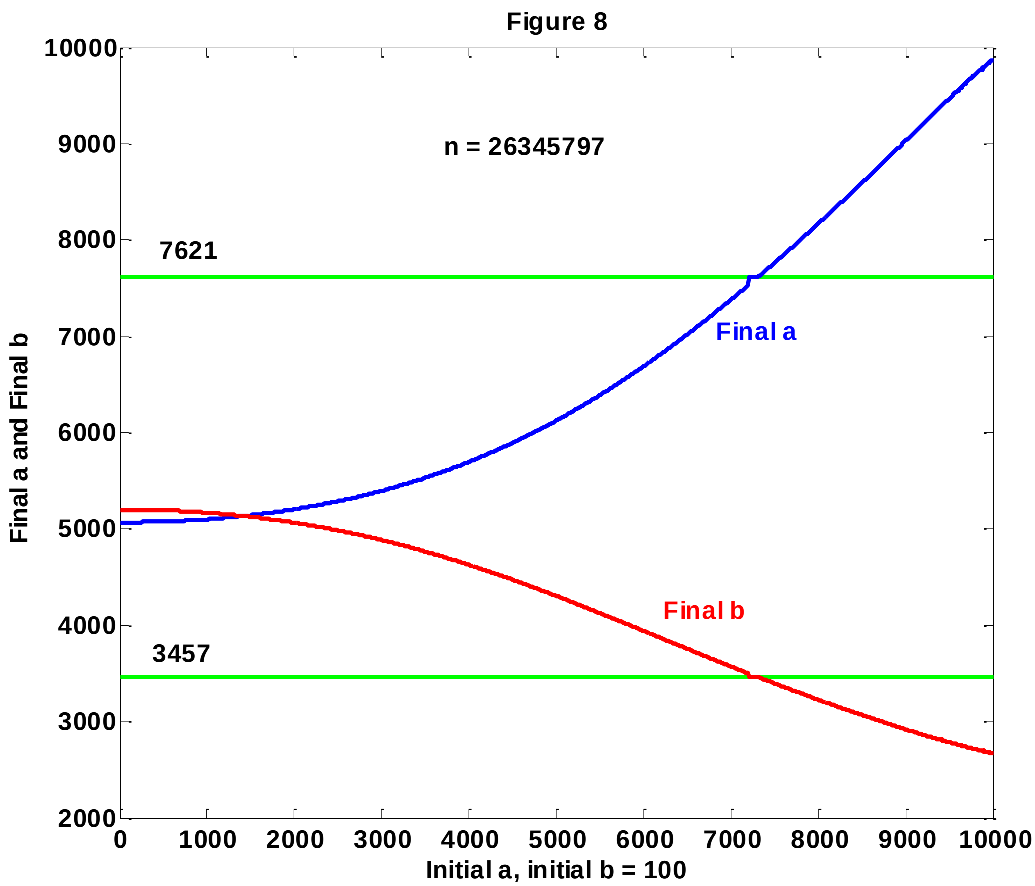






As can be seen slight changes in the original number result in significant changes to where the step is seen. In Figure 7d the step is not seen at all. Perhaps in Figure 7d the $initial\ a$ could be brought out further, say to $initial\ a = 20000.$ One will find that at $initial\ a = 10621$ the b term becomes complex and the feedback process blows up. What accounts for this mysterious step anomaly in the curves where it is seen?

An answer can be found in Figure 8. The green lines are the two factors of $n = 26345797$.



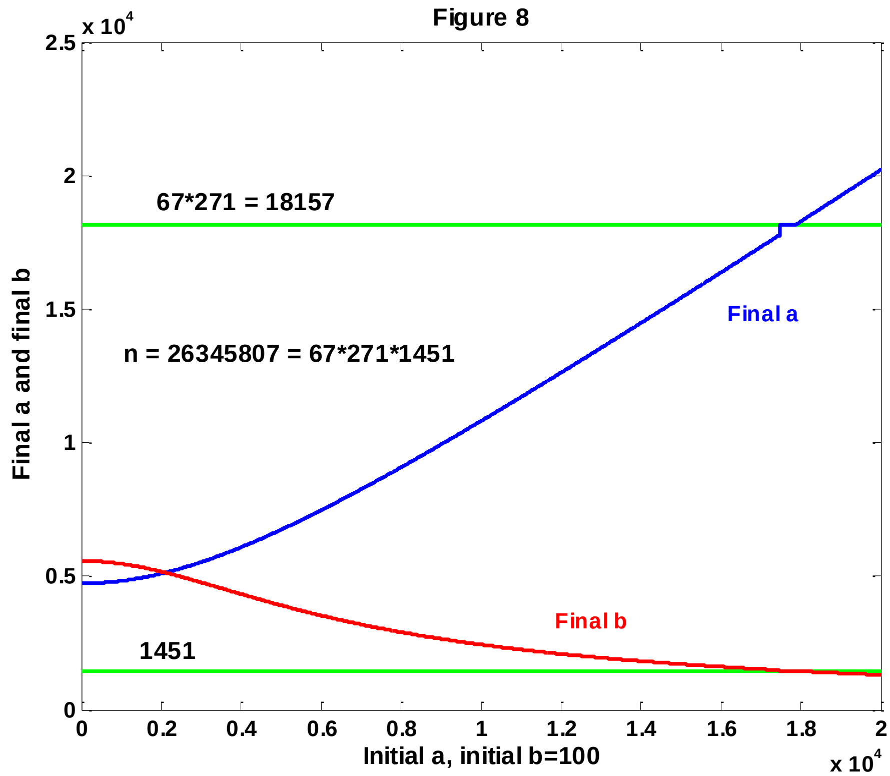
The steps in the curves are at the factors of the n in the feedback process. In figures 7b and 7c the steps are at the factors of the n in the process or combinations of those factors. For the n in Figure 7d, $n = 26345797 + 10$ and the factors are $67,\ 271\ and\ 1451$. The factors and any combination of them are outside the limits of $final\ a\ \ or\ \ final\ b$ owing to b becoming complex as $initial\ a$ passes a certain value.

While there are more efficient methods of determining the factors of a number in the lower hills of the math world, the landscape one stumbles on here is more picturest. The process is not trial and error. It can lead directly to a factor, although the process is long. The mathematical operations used are multiplication, subtraction and square root. No operations of division are made. If $ceil\ and\ floor$ in *FB1* are replaced with any other combination of $ceil,\ floor\ or\ round$ the process does not show a step at the factors. The process is limited to where b becomes complex. If that could be avoided, it would be more universal. Since factors are of concern, one may try replacing the last term in the feedback loop with $b = ceil(n/a)$. The process becomes

$$FB2(feedback\ 2)$$

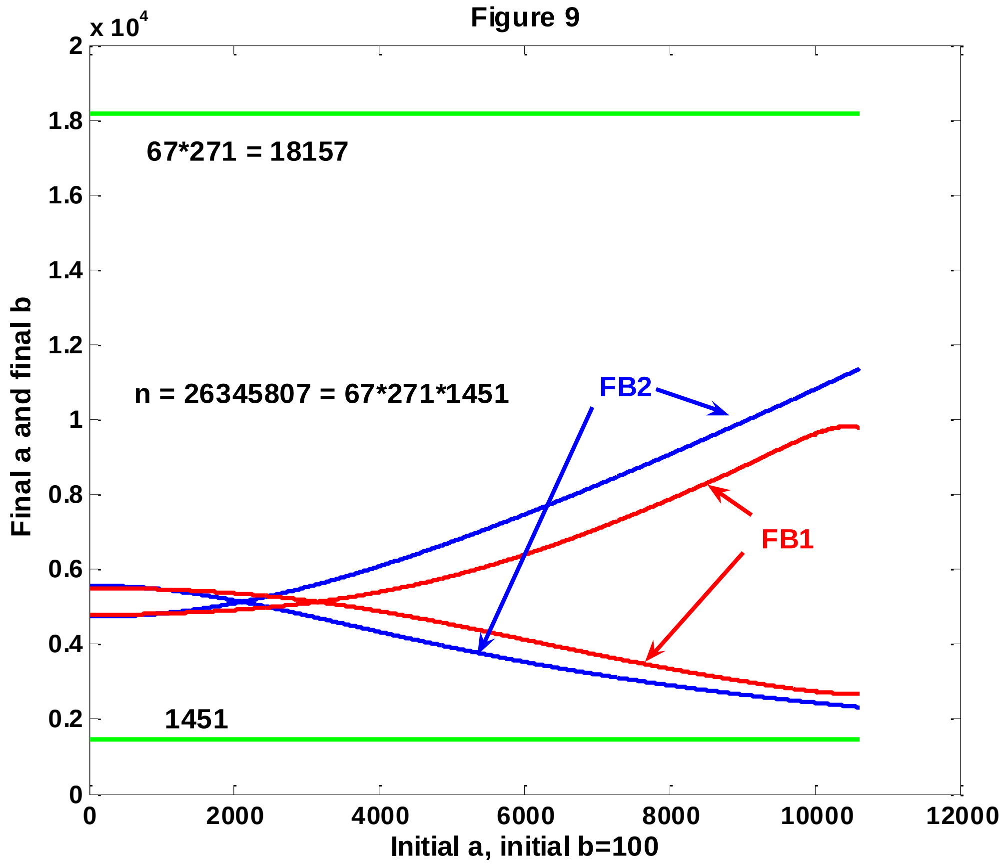
$$\begin{array}{l} \rightarrow\rightarrow a = floor\left(\sqrt{n - a(b - a)}\right) \\ \uparrow \quad\quad \downarrow \\ \leftarrow\leftarrow b = ceil(n/a) \end{array}$$

Now b will be complex only if a is complex. Using this feedback process for $n = 26345797 + 10$ one can now see the factors or combination of factors for n as shown in Figure 8.



The largest factor is seen in $final\ b$ and the combination of the two smallest factors is seen in $final\ a$.

Before leaving this hike in the lower hills of the math world, one must investigate any difference in $final\ b$ between $FB1$ and $FB2$. The difference up to where $final\ b$ becomes complex is shown in figure 9.



Although the feedback loops give similar results up to where $FB1$ becomes complex, they are not identical. The results for $FB2$ will continue to be non-complex numbers and will form a step at the factors of n or combination of factors as shown by the green lines.

This has been as interesting hike through the lower math terrain. The time is getting late and the sun is going down. The day hike in the lower hills of the math world must come to an end. Terrain was encountered that was new to mathdroolers. Only a small part of the territory was explored. There may be much more, which will have to wait for another day.