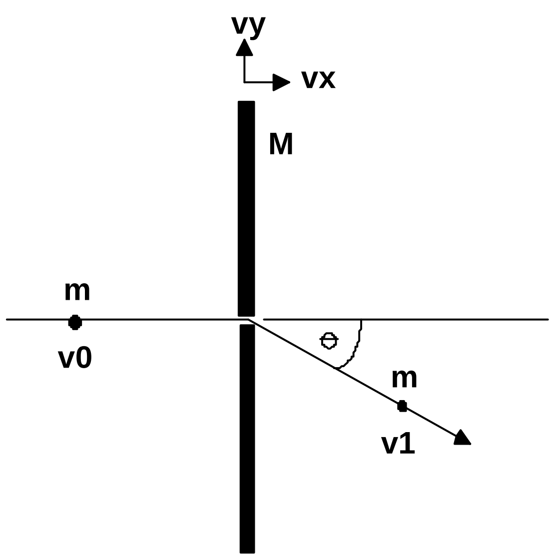
Quantum Fantasy

Sometimes when hiking the lower hills of the math world one finds themselves in the outskirts of the physics world. The landscape today consists of a particle and a wall with a slit in it. The particle is in motion toward the wall and finds its way through the slit. It comes out on the other side of the wall not necessarily in the same direction it was moving and most likely in a different direction. This is called diffraction through a slit. The wall will be considered to be in free space free floating. Being in the lower hills of the math world one can take liberties with reality for fun and profit.

The situation being observed is pictured below.



The only two entities are the wall with the slit and the particle. It will also be assumed a third entity is a universe for which a reference point somewhere has been chosen to measure to. The interaction of the wall and the particle will be made using what was learned in first year physics, i.e. classical physics. The two relations between interacting entities are the conservation of energy and the conservation of momentum.

Momentum:

x-direction $mv_0 = mv_1\cos(\theta) + MV_x \implies V_x = \frac{1}{\eta}(v_0 - v - 1\cos(\theta))$ where $\eta = \frac{M}{n}$

y-direction $0 = MV_y = mv_1\sin(\theta) \implies V + y = \frac{1}{\eta}v_1\sin(\theta)$

Energy
$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}M\left(\sqrt{V_x^2 + V_y^2}\right)$$
$$\implies \frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}M\left[\frac{1}{\eta^2}(v_0 - v_1\cos(\theta))^2 + \frac{1}{\eta^2}v_1^2\sin^2(\theta)\right]$$
$$\implies v_0^2 - v_1^2 = \eta\left(\frac{1}{\eta^2}\right)[v_0^2 - 2v_0v_1\cos(\theta) + v_1^2\cos^2(\theta) + v_1^2\sin^2(\theta)$$
$$\implies 0 = (1-\eta)v_1^2 - 2v_0v_1\cos(\theta) + (1-\eta)v_0^2$$
$$\implies v_1 = \frac{2v_0\cos(\theta) \pm \sqrt{4v_0^2\cos^2(\theta) - 4(1+\eta)(1-\eta)v_0^2}}{2(1+\eta)}$$
$$\implies v_1 = v_0\left[\frac{\cos(\theta) \pm \sqrt{\eta^2 - \sin^2(\theta)}}{1+\eta}\right] \quad (1)$$

The mass of the wall being much greater than the mass of the particle results in

$$M \gg m$$
$$\implies \eta \gg 1$$
$$\implies v_1 = v_0$$

This hike is not in the hills of the physics world, it is in the lower hills of the math world. As said above, liberties can be taken. What if the mass of the wall is not much greater than the mass of the particle? What if the mass of the wall goes to zero?

Equation 1 then becomes

$$v_1 = v_0e^{\pm i\theta} \quad (2)$$

Starting with classical entities and classical notions the final motion is complex. Part of the landscape was also diffraction through a slit. Only the barest concept of diffraction was used, which was that the particle changed direction on leaving the slit. Momentum and energy were transferred between the particle and the slit by means other than any of the known forces.

Looking at Equation 2 it could be thought that in some way the particle has a wave nature attribute. That may be a stretch to conclude. Diffraction was one of the starting points and is a result of the particle having a wave nature. However, no details of diffraction were used. What was used was that the particle changed direction on leaving the slit and that momentum and energy were conserved. The non-reality of the mass of the wall being set to zero was applied and the strange result followed.

As was done with a particle can be done with photons. Below is the math without comment. Reference can be made to the diagram above where now ν is frequency instead of the velocity v. The symbols h and c are Plank's constant and the speed of light in a vacuum, respectively.

Momentum:

x-direction $\quad \frac{h\nu_0}{c} = \frac{h\nu_1 \cos(\theta)}{c} + MV_x \quad \Longrightarrow V_x = \frac{h}{cM}(\nu_0 - \nu_1 \cos(\theta))$

y-direction $\quad 0 = \frac{h\nu_1 \sin(\theta)}{c} - MV_y \Longrightarrow V_y = \frac{h}{cM}\nu_1 \sin(\theta)$

Energy $\quad h\nu_0 = h\nu_1 + \frac{M}{2}(V_x^2 + V_y^2)$

$$= h\nu_1 + \frac{M}{2}\frac{h^2}{c^2M^2}[\nu_0^2 - 2\nu_0\nu_1\cos(\theta) + \nu_1^2\cos^2(\theta) + \nu_1^2\sin^2(\theta)]$$

$$\Longrightarrow \nu_0 - \nu_1 = \frac{1}{\eta}[\nu_0^2 - 2\nu_0\nu_1\cos(\theta) + \nu_1^2] \qquad \left(= \frac{1}{\eta}|\overrightarrow{\nu_0} \circ \overrightarrow{\nu_1}|\right) \text{ where } \eta = \frac{Mc^2}{h}$$

$$\Longrightarrow 0 = \nu_1^2 + (2\eta - \nu_0\cos(\theta))\nu_1 + (\nu_0^2 - 2\eta\nu_0)$$

$$\Longrightarrow \nu_1 = -\eta + \nu_0\cos(\theta) \pm \sqrt{\eta^2 - 2\eta\nu_0\cos(\theta) + \nu_0^2\cos^2(\theta) - \nu_0^2 + 2\eta\nu_0}$$

$$\Longrightarrow \nu_1 = -\eta + \nu_0\cos(\theta) \pm \sqrt{\eta^2 - 2\eta\nu_0(1 - \cos(\theta)) - \nu_1^2\cos^2(\theta)}$$

As was done above set $M = 0$ which yields $\eta = 0$ giving

$$\nu_1 = \nu_0\cos(\theta) \pm i\nu_0\sin(\theta)$$

$$\nu_1 = \nu_0 e^{\pm i\theta}$$

Math Drooler 16Feb11
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